

# Model for Road Network Stochastic User Equilibrium Based on Bi-level Programming under the Action of the Traffic Flow Guidance System

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**Abstract:** With application of the traffic flow guidance system (TFGS), some travelers can choose their paths according to both the guidance information and their experience. In this paper, the benefit-cost of the traffic flow guidance system and the random users' equilibrium under the action of the traffic flow guidance system are combined. Based on the relationship between the effects of traffic guidance and the construction cost, the optimization plan is established. The relationship between the optimization system and the random users under the action of the traffic flow guidance system is described using the bi-level programming system, and an arithmetic based on the sensitivity analysis is presented.

**Key Words:** bi-level programming; traffic flow guidance; stochastic user equilibrium; prospect theory

## 1 Introduction

With the development of mechanization in metropolis, the traffic conditions are becoming increasing worse. Intelligent Traffic System (ITS) has become the effective approach to solve traffic problems in cities. As the important subsystem of ITS, the Traffic Flow Guidance System (TFGS) has enriched the road network information that travelers can acquire, and this allows the travelers to make more reasonable route choices. At the same time, TFGS has changed the route-choice behaviors of travelers and the equilibrium of the network. Under the action of the TFGS, a new explanation to the Stochastic User Equilibrium (SUE) based on the Prospect Theory (PT) is brought forward and a better route-choice model that can reflect the fact of the user's route choice behavior more effectively is established in the first reference.

With deeper study and application of TFGS, domestic and foreign experts have realized gradually that the application of TFGS may cause some bad effect. Some experts found that one bad phenomenon will be induced by the application of TFGS, that is, at the same time, several travelers may be influenced by the guidance information and they may choose the same path at that moment<sup>[2–4]</sup>. The result is that the

expedite path may become jammed, and traffic confusions may occur<sup>[2,4]</sup>. To avoid such a situation, traffic governors expect that with the application of TFGS, the overall delay of the traffic system will reduce to a great extent, and the cost of the system will not exceed the budget. As for the travelers, they will choose the best path for their own benefit according to the traffic information that they have mastered.

Since the goal between the upper-level governors and the lower-level travelers is different, this leads to the situation that when the travelers choose their path according to their guidance information, and the state of the road network reaches equilibrium, the state is not always the best optimization as expected by the upper-level governors. A bi-level programming model is established in this paper to describe the urban traffic network equilibrium problem under the action of TFGS. On one hand, from the point of view of the travelers, their behaviors follow the rule of SUE; on the other hand, from the point of view of the system (the governors), the summation of the total impedance of the system and the investment of the guidance system must be minimum, or the total impedance of the system must be minimally subjected to the limit of the investment.

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## 2 Introduction of the bi-level programming model

The leader-follower hierarchical decision problem was first put forward in 1952 by Stackelberg when he studied the market economy problem, which is the so-called bi-level programming problem. Bi-level programming problem is the special case of multi-level programming. The decision maker of each level is independent correspondingly, that is, the upper-level decision maker only directs (or leads) the lower-level decision maker to make his own decision, and does not intervene the decision of the lower-level directly; accordingly, the lower-level decision maker takes the decision of the upper-level as the parameter or restriction only, and he can make his own decision freely in his range<sup>[5,6]</sup>.

The decision process of bi-level programming is as follows: the upper-level provides the lower-level with some information, and the lower-level takes the decision according to his own benefit or favor; the upper-level will make the decision, which will meet the global benefits according to the reactions. The information provided by the upper-level is given as a kind of decision. Actually, the lower-level action is the countermeasure to the upper-level decision. It is the best countermeasure to the lower-level and is relevant to the upper-level information. To earn the best benefit for the overall system, the upper-level must adjust its countermeasure with the combination of the lower-level decision<sup>[6]</sup>.

Generally, the form of the bi-level programming is as follows<sup>[5–7]</sup>:

$$(U) \quad \min_x F(x, y) \quad (1)$$

$$\text{s.t.} \quad G(x, y) \leq 0 \quad (2)$$

where  $y = y(x)$  can be concluded by the following programming:

$$(L) \quad \min_x f(x, y) \quad (3)$$

$$\text{s.t.} \quad g(x, y) \leq 0 \quad (4)$$

where  $F: E^{n1} \times E^{n2} \rightarrow E^1$ ,  $G: E^{n1} \times E^{n2} \rightarrow E^{m1}$ ,  $f: E^{n1} \times E^{n2} \rightarrow E^1$ ,  $g: E^{n1} \times E^{n2} \rightarrow E^{m2}$ .

We can conclude from the bi-level programming model that this model is composed of the upper-level programming model (U) and the lower-level programming model (L). The upper-level decision-maker affects the lower-level decision maker by the value setting of  $x$ , and thus, this limits the feasible restrict collection. The upper-level decision-maker interacts with the lower-level decision-maker by the destination function of the lower-level decision-maker. The lower-level decision variable  $y$  is the function of the upper-level decision variable  $x$ , that is,  $y = y(x)$ . This function is called the response function.

As compared with the mono-level programming, the bi-level programming has the unexampled merit. The detailed representations are: (i) The bi-level programming can analyze

two different ambivalent targets in the decision-making progress; (ii) The multi-value rules of the bi-level programming are closer to the fact; (iii) The bi-level programming can express the interaction of the government and the public definitely<sup>[7]</sup>.

## 3 Model for road network SUE based on the bi-level programming model under the action of TFGS

Before the optimized model is established, the related symbols are defined first.

### 3.1 Symbol definitions

$N$ : node (index) set;

$A$ : arc (index) set;

$R$ : set of origin nodes,  $R \in N$ ;

$F$ : set of destination nodes,  $F \in N$ ;

$r$ : the origin node,  $r \in R$ ;

$s$ : the destination node,  $s \in F$ ;

$K_{rs}$ : the set of paths between  $r$  and  $s$ ;

$q_{rs}$ : the total traffic demand between origin  $r$  and destination  $s$ ;

$x_a$ : the total flow on arc  $a$ ;

$j_k$ : flow on path  $k$  connecting O-D pair  $r-s$ ;

$f_{rs}$ : vector  $(\dots, f_k^{rs}, \dots)$ ,  $k \in K_{rs}$ ;

$\beta_p$ : parameter of the model;

$PT_k^{rs}$ : the prospect-theoretic value of path  $k$  connecting O-D pair  $r-s$ ,  $k \in K_{rs}$ ;

$PT_a$ : the prospect-theoretic value of arc  $a$ ,  $a \in A$ ;

$\delta_{a,k}^{rs}$ : indicator variable,

$$\delta_{a,k}^{rs} = \begin{cases} 1 & \text{if link } a \text{ is on path } k \text{ between O-D pair } r-s \\ 0 & \text{otherwise} \end{cases}$$

The relationship between the path flow and the link flow, and the relationship between the path prospect-theoretic value and the link prospect-theoretic value can be expressed mathematically as:

$$PT_k^{rs} = \sum_a PT_a \delta_{a,k}^{rs} \quad \forall k \in K_{rs}, \forall r \in R, \forall s \in F \quad (5)$$

$$x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A \quad (6)$$

### 3.2 The upper-level programming problem: system optimum of the road network

From the point of the upper-level governor, the final purpose of the establishment of the TFGS is to optimize the capability of the whole network, that is, to minimize the summation of the total impedance of the system and the investment of the guidance system, or minimize the total impedance of the system subjected to the restriction of the investment. Thus, there are two upper-level programming models:

(i) Taking no account of the restriction of the investment, the system optimum can be scaled by two aspects: the total

delay of the network, and the investment of the TFSGS. The total delay of the network can be denoted by  $F_1$ :  $F_1 = \sum_a x_a t_a(x_a)$ , and the investment of the TFSGS can be denoted by  $F_2$ :  $F_2 = \sum_a h_a(\beta_p)$ . Generally, these two indexes are not optimum at the same time; thus, without regard to the restriction of the investment, the upper-level programming can be expressed by the summation of the generalized cost of the two indexes, that is, minimum  $F = F_1 + \rho F_2$ , where  $\rho$  is the conversion coefficient. The upper-level programming model can be established as:

$$(U1) \quad \min F = \sum_a x_a(\beta_p) t_a(x_a(\beta_p)) + \rho \sum_a h_a(\beta_p) \quad (7)$$

$$\text{s.t.} \quad \sum_k f_k^{rs} = q_{rs} \quad \forall r, s \quad (8)$$

$$f_k^{rs} \geq 0 \quad \forall k, r, s \quad (9)$$

$$x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A \quad (10)$$

(ii) Taking account of the restriction of the investment, the system optimum can be scaled by the total delay of the network. The upper-level programming model can be established as:

$$(U2) \quad \min F = \sum_a x_a(\beta_p) t_a(x_a(\beta_p)) \quad (11)$$

$$\text{s.t.} \quad \sum_k f_k^{rs} = q_{rs} \quad \forall r, s \quad (12)$$

$$f_k^{rs} \geq 0 \quad \forall k, r, s \quad (13)$$

$$x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A \quad (14)$$

$$\sum_a h_a(\beta_p) \leq H \quad (15)$$

where  $H$  is the total investment of the TFSGS.

### 3.3 Lower-level programming problem: the SUE under the action of TFSGS

The SUE under the action of TFSGS is different from the normal user equilibrium. To describe the SUE under the action of TFSGS exactly, the prospect theory is applied to the route choice.

#### 3.3.1 The route choice function of applying the prospect theory

Based on the first Wardrop equilibrium theory (User Equilibrium), the travel time of those paths carrying flow is equal to each other, which are less than the travel time of those paths not carrying flow. Thus, the selection probability of the route is:

$$P_j = P(C_j^{rs} \leq C_k^{rs}, \forall k \neq j) \quad (16)$$

This probability of route choice is based on two assumptions, one of which is that all travelers try their best to choose the route with the least impedance; the second is that all travelers know the condition of the route network at any moment and can make the correct route choice. In fact, the assumptions are impossible in reality.

In reality, according to the character of the traveler's route

choice, travelers can be classified into three kinds: fixed route travelers (eg., bus drivers, periodic travelers), the commuters, and one-time travelers. Under the action of TFSGS, different kinds of travelers have different route-choice characteristics, and all the characteristics can be explained by PT<sup>[1]</sup>.

(i) Periodic travelers may change route in special conditions. For example, office workers usually do not change their route, even if they receive the traffic guidance information. However, if they are sure that they cannot reach their offices on time by the fixed route, and the guidance information shows that another route expedites their trip destinations, they may accept the guidance and change their route even if the traffic flow guidance information has an error.

This phenomenon can be interpreted by the nature of the value function (Eq. (26)), that is, when people confront gains, they tend to elude risks, but when they confront losses, they will prefer risks. Under normal road conditions, travelers will not be late by choosing the fixed route. They will not like to take a risk to change their normal route, even if another route is faster in some days. However, if they are sure that they cannot reach their office on time, they will choose another route with a chance attitude.

(ii) As the route choice shown in Figures 1 and 2, assuming that the information of the traffic flow guidance system is time. In Figure 1, most travelers will choose route 1. The probability of choosing route 1 or route 2 in Fig. 2 is almost the same.

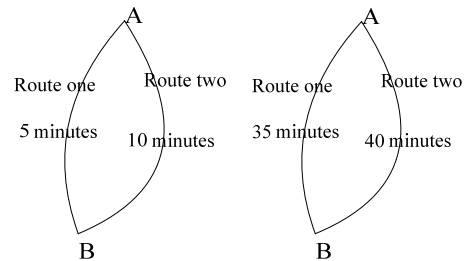


Fig. 1 Gain of route choice 1

Fig. 2 Gain of route choice 2

This phenomenon can be explained by the character of the value function (Eq. (26)), that is, people have different attitudes to gains with different reference points. The gain of both route choice 1 and 2 is 5 min in the above Figs. But in Fig. 1, the deviation to the reference point is lower than that in Fig. 2, and therefore, the behavior of the route choice is different.

(iii) Small errors in the information will influence the effect of the traffic flow guidance system seriously. As the system is influenced by several factors, it is impossible to forecast the information absolutely accurately, that degrades the trust degree of the people. If inaccurate information is yielded 15 out of 100 times, the trust degree of travelers to this system is lower than 85%.

The nature of the value function (Eq. (26)) can be used to explain this phenomenon. Under the circumstance of the same gain and loss, marginal losses are more sensitive than marginal gains, and the pain of losses is more than the joy of gains. People will always exaggerate the influence of the 15 times inaccurate information, and the trust in the system goes down. This phenomenon can also be explained by the weighting function (Eq. (27)), that is, individuals will excessively concern themselves with the low probability affairs. The inaccuracy is the low probability affair, but people are excessively concerned about it, and the trust degree becomes lower than what was thought earlier.

Now that the prospect theory can adequately explain the route choice behavior under the action of TFGS, and the prospect-theoretic value can be used as the basis of the route choice, travelers will choose the route with the largest prospect-theoretic value  $PT^{[1]}$ , and thus, the possibility of the route that is chosen can be expressed as:

$$P_j = P(PT_j^{rs} \geq PT_k^{rs}, \forall k \neq j) \quad (17)$$

Since several factors can influence the prospect-theoretic value, different travelers have different knowledge about the route, and different travelers have different reference points; thus different travelers have different prospect-theoretic values. To incorporate the effects of the unobserved factors and characteristics, the prospect-theoretic value must be expressed as a random variable, and it consists of a systematic (deterministic) component and an additive random error term, that is:

$$PT_k = pt_k + \xi_k \quad \forall k \in K \quad (18)$$

where  $pt_k$  is the systematic component and  $\xi_k$  is the random error term. The expectation of the random error is 0, that is,  $E(PT_k) = pt_k$ .  $PT_k$  denotes the perceived utility indicating that the decision-maker perceives the utility of route  $k$  as  $PT_k$ , and  $pt_k$  denotes the measured utility indicating that the system analyst measured the utility of route  $k$  as  $pt_k$ .

Assuming that the random error terms  $\xi_k = PT_k - pt_k \quad \forall k \in K$  of each utility function are independently and identically obey to Gumbel distribution, the density function can be expressed as:

$$\phi(x) = \beta_p e^{-\beta_p x + E} e^{-e^{-\beta_p x + E}}$$

where  $E$  is Euler's constant (eg.,  $E=0.5708\dots$ ), and  $\beta_p$  is a parameter of the function.

Then, the probability  $P_j$  can be given by Eqs. (19) and (20)<sup>[8]</sup>.

$$P_j = \frac{e^{\beta_p PT_j^{rs}}}{\sum_k e^{\beta_p PT_k^{rs}}} \quad (19)$$

$$f^{rs} = q_{rs} \frac{e^{\beta_p PT_j^{rs}}}{\sum_k e^{\beta_p PT_k^{rs}}} \quad (20)$$

This is the logic model. Since the utility value is the prospect-theoretic value, the behavior of route choice can be described more fittingly for the reality with TFGS.

### 3.3.2 Equivalent model

Based on the analysis of the application that the prospect theory is used in the route choice, the equivalent model can be established as below, and this model is used as the lower level programming model:

$$(L) \quad \min Z(f) = \frac{1}{\beta_p} \sum_r \sum_s \sum_k f_k^{rs} \ln f_k^{rs} - \sum_a \int_0^{x_a} PT(w) dw \quad (21)$$

$$\text{s.t.} \quad \sum_k f_k^{rs} = q_{rs} \quad \forall r, s \quad (22)$$

$$f_k^{rs} \geq 0 \quad \forall k, r, s \quad (23)$$

$$x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A \quad (24)$$

In the objective function,  $PT(x)$  is the prospect-theoretic value calculated by the following formula<sup>[9-11]</sup>. Provided that the probability of  $x_i$  is  $p_i$  in the course of choosing route  $i$ , the prospect-theoretic value can be expressed as:

$$PT = \sum_k \pi(p_i) v(x_i) \quad (25)$$

where  $v(\bullet)$  is the value function, and  $\pi(\bullet)$  is a probability weighting function.

The value function  $v(\bullet)$  given by Tversky and Kahneman is:

$$v(x) = \begin{cases} x^\alpha & x \geq 0 \\ -\lambda(-x)^\beta & x < 0 \end{cases} \quad (26)$$

where  $\lambda \geq 1$ , this parameter describes the degree of loss aversion;  $\alpha, \beta \leq 1$  measures the degree of diminishing sensitivity.

The weighting functions that given by Tversky and Kahneman are:

$$\begin{cases} \pi^+(p) = p^\gamma / [p^\gamma + (1-p)^\gamma]^{1/\gamma} & \text{losses} \\ \pi^-(p) = p^\delta / [p^\delta + (1-p)^\delta]^{1/\delta} & \text{gains} \end{cases} \quad (27)$$

Tversky and Kahneman estimated the parameters in Eqs. (26) and (27), and the results are:  $\alpha=\beta=0.88$ ;  $\lambda=2.25$ ;  $\gamma=0.61$ ;  $\delta=0.69$ .

### 3.4 Model for road network SUE based on the bi-level programming model under the action of TFGS

By summarizing the former discussion and by combining the upper-level programming model and the lower-level programming model, the bi-level programming model can be established.

#### 3.4.1 Bi-level programming model 1

$$(U1) \quad \min F = \sum_a x_a(\beta_p) t_a(x_a(\beta_p)) + \rho \sum_a h_a(\beta_p)$$

$$\text{s.t.} \quad \sum_k f_k^{rs} = q_{rs} \quad \forall r, s$$

$$f_k^{rs} \geq 0 \quad \forall k, r, s$$

$$x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A$$

where  $x_a(\beta_p)$  can be calculated by the lower-level programming model:

$$\begin{aligned}
\text{(L)} \quad \min Z(f) &= \frac{1}{\beta_p} \sum_r \sum_s \sum_k f_k^{rs} \ln f_k^{rs} - \sum_a \int_0^{x_a} PT(w) dw \\
\text{s.t.} \quad &\sum_k f_k^{rs} = q_{rs} \quad \forall r, s \\
&f_k^{rs} \geq 0 \quad \forall k, r, s \\
&x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A
\end{aligned}$$

### 3.4.2 Bi-level programming model 2

$$\begin{aligned}
\text{(U2)} \quad \min F &= \sum_a x_a(\beta_p) t_a(x_a(\beta_p)) \\
\text{s.t.} \quad &\sum_k f_k^{rs} = q_{rs} \quad \forall r, s \\
&f_k^{rs} \geq 0 \quad \forall k, r, s \\
&x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A \\
&\sum_a h_a(\beta_p) \leq H
\end{aligned}$$

where  $x_a(\beta_p)$  can be calculated by the lower-level programming model:

$$\begin{aligned}
\text{(L)} \quad \min Z(f) &= \frac{1}{\beta_p} \sum_r \sum_s \sum_k f_k^{rs} \ln f_k^{rs} - \sum_a \int_0^{x_a} PT(w) dw \\
\text{s.t.} \quad &\sum_k f_k^{rs} = q_{rs} \quad \forall r, s \\
&f_k^{rs} \geq 0 \quad \forall k, r, s \\
&x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A
\end{aligned}$$

### 3.4.3 Explanation to the model

In the model, the stochastic character of the model is controlled by  $\beta_p$  which represents the perceiving degree to the road network of drivers. The deeper the perceiving degree to the road network of drivers, the smaller is the impedance variance of the network, and the bigger is the value of  $\beta_p$ . The purpose of the TFGS establishment is to adjust the perceiving degree to the network, that is, to adjust the drivers' understanding to the route impedance and to increase the value of  $\beta_p$  sequentially. When  $\beta_p$  reaches infinite, the model is changed into standard UE based on the prospect theory. However, from the point of view of the traffic governors, since the total delay of the system is not always the minimum when the system reaches UE, they do not hope to reach the user equilibrium, but hope to reach the system optimum, that is, they do not hope that the value of  $\beta_p$  reaches infinite. This determines that the TFGS investment must be moderate, and thus  $\beta_p$  is adjusted to optimize the whole system.

The difference of the two models is the upper-level programming model. The upper-level programming model of the first model has no restriction of total investment, that is, regardless of how much you invest, the whole effect of the system must be the best in the end. This model can be taken as a reference by governors in the long term investment and the final effect that the TFGS can reach in the future can be

mastered without considering the investment. The upper-level programming model of the second model has the restriction of total investment, that is, the whole delay of the system is minimally subject to the restriction of the investment. This model can be taken as a reference of the phrase investment to build the traffic flow guidance system in short term.

If the value of the conversion factor  $\rho$  in model 1 (Eq. (7)) is equal to the Lagrange multipliers of the investment budget constraint (Eq. (15)) in model 2, the two models are exactly the same. The reason why the two models are used is entirely for the sake of calculation and application.

## 4 The algorithm of the bi-level programming model

Generally, bi-level programming problem is an NP-hard problem and has no polynomial algorithm, and therefore, the solution of the bi-level programming is very complicated, and sometimes, the result is not the global best solution but the local best solution. Commonly, heuristic algorithm is applied to solve the bi-level programming problem. In this paper, the algorithm based on the sensitivity analysis method is used<sup>[6,7]</sup>.

The algorithm of the upper-level programming can be summarized as follows:

Step 1: set  $\beta_p$  as the initial value and set iterative times  $k=0$ .

Step2: according to  $\beta_p^k$ , compute the lower-level programming problem and yield a link-flow  $x^k$ .

Step 3: compute the derivative of balanceable link-flow  $x^k$  to  $\beta_p^k$  by the sensitivity analysis method.

Step 4: calculate Eq. (28), and then replace the result in the upper-level target function, and calculate the upper-level programming problem and yield a new  $\beta_p^{k+1}$ .

Suppose  $\beta_p'$  is the initialized value,  $x_a(\beta_p')$  is the corresponding balanceable link-flow (yield by the lower-level programming problem), then

$$x_a(\beta_p) \approx x_a(\beta_p') + \sum \left[ \frac{\partial x_a(\beta_p)}{\partial \beta_p} \right]_{\beta_p = \beta_p'} (\beta_p - \beta_p') \quad (28)$$

Step 5: if  $|\beta_p^{k+1} - \beta_p^k| \leq \sigma$ , then stop (where  $\sigma$  is the precision of iterative); otherwise, set  $k=k+1$  and go to step 2.

The algorithm of the lower-level programming can be summarized as follows:

Step1: initialization, find a set of valid routes.

(i) Compute the minimum impedance (the prospect theory value) from the origin  $r$  to all other nodes. Determine  $r(i)$  for each node  $i$ .

(ii) Compute the minimum impedance (the prospect theory value) from each node to final node  $s$ . Determine  $S(i)$  for each node  $i$ .

(iii) Define  $Q_i$  as the set of downstream nodes of all links leaving node  $i$ .

(iv) Define  $D_i$  as the set of up stream nodes of all links arriving at node  $i$ .

(v) For each link  $(i, j)$ , compute the “link likelihood” (suppose parameter  $b=1$ ).

$$L(i, j) = \begin{cases} \exp\{\beta_p[r(j) - r(i) - PT(i, j)]\}, & \text{if } r(i) < r(j) \text{ and } s(i) > s(j) \\ 0, & \text{otherwise} \end{cases}$$

All links that the likelihood is equal to 0 are not reasonable links, and thus the path including these links must be excluded and not taken into account. All links that the likelihood is larger than 0 can be taken into account in the valid path. If the likelihood of all links is equal to 1 in some paths, the paths must have the smallest impedance.

Step 2: compute the impedance of the valid route when the flow is 0, that is, to compute the PT value of all the valid routes when the flow is 0, and then calculate the path flow  $f_{k,n}^{rs}$ ,  $\forall r, s, k$  according to Eq. (20); set  $n=1$ .

Step 3: compute the new PT value according to  $f_{k,n}^{rs}$ ,  $\forall r, s, k$ , and compute the new path flow  $h_{k,n}^{rs}$  according to Eq. (20).

Step 4:  $0 \leq b \leq 1$ , set  $f_{k,n+1}^{rs} = (1-b)f_{k,n}^{rs} + bh_{k,n}^{rs}$ ,  $\forall r, s, k$ ; compute the one dimension hunting problem as follows and decide the iterative step  $b_n = b^*$ .

$$\min Z(b) = \frac{1}{\beta_p} \sum_r \sum_s \sum_k f_{k,n+1}^{rs} \ln f_{k,n+1}^{rs} - \sum_a \int_0^{x_a^{n+1}} PT(w) dw \quad (29)$$

$$\text{where } x_a^{n+1} = \sum_r \sum_s \sum_k f_{k,n+1}^{rs} \delta_{a,k}^{rs} \quad \forall a \in A$$

The path flow can be updated as

$$f_{k,n+1}^{rs} = (1-b_n)f_{k,n}^{rs} + b_n h_{k,n}^{rs}, \quad \forall r, s, k$$

Step 5: convergence test. If the result is satisfactory, stop; otherwise, set  $n=n+1$  and go to step 3.

## 5 Conclusions

The route choice behaviors of travelers change accordingly under the action of TFGS. At the same time, the prospect theory can interpret this change felicitously and can be applied to interpret the route choice behavior of the travelers more reasonably. In this paper, based on the application of the prospect theory to the route choice, the route network stochastic user equilibrium under the action of TFGS is studied, and a bi-level programming of establishing the TFGS is applied with or without the restriction of investment and an algorithm is put forward. In this model, the hopes of the governors and the travelers are combined skillfully, the immoderate establishment can be avoided and the bad effect

can be avoided accordingly. There are several problems that needed to be studied in the following task, such as the relationship between parameter  $\beta_p$  and the building expenditure, and the method to calibrate  $\beta_p$  and parameter  $\alpha, \beta, \lambda, \gamma, \delta$ , appropriate to the route choice. With deeper study of the task, another paper will be written to discuss these problems.

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